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TENSILE PROPERTIES OF A TAPE

WITH A TRANSVERSE CREASE

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TENSILE PROPERTIES OF A TAPE WITH A TRANSVERSE CREASE

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SUMMARY

A theory is developed for the shortening of a metallic tape by a transverse crease when the tape is carrying a longitudinal tensile load. It is assumed that a plastic hinge develops at the crease if sufficient stress occurs at that point. The effective tensile modulus of a creased tape is derived.

INTRODUCTION

Current interest in the design of very large reflectors for orbiting radiotelescopes operating at low frequencies has led to the investigation of methods whereby the surface density of radio reflectors can be made as low as possible. This need for lightness, coupled with the requirements for reflectivity, for operation in a micrometeoroid environment, and for maintaining a geometrically precise surface, has led the Astro Research Corporation to the investigation of networks of ribbons as radio reflectors.

The placing into orbit of a very large radiotelescope requires that the system somehow be folded into a relatively small volume during the launch phase. Although it may be feasible to fold a network of ribbons without any of the elements being creased or otherwise deformed beyond the elastic limit, the acceptance of such creases may greatly simplify the packaging problems.

Since the static shape of the network and the effective tensile modulus of the ribbon elements, and thus the vibratory frequencies of the system, will depend upon the behavior of the ribbons in the vicinity of creases, a study of such behavior has been made and is reported herein.

THE SHAPE OF THE CREASED TAPE

A very long metallic tape is assumed to have a tight transverse crease placed in it and then be subjected to a straightening force at the ends. For forces below some level, the material will all be stressed within the elastic range. For higher forces, plastic deformation will occur in the general region of the crease as shown in figure 1.

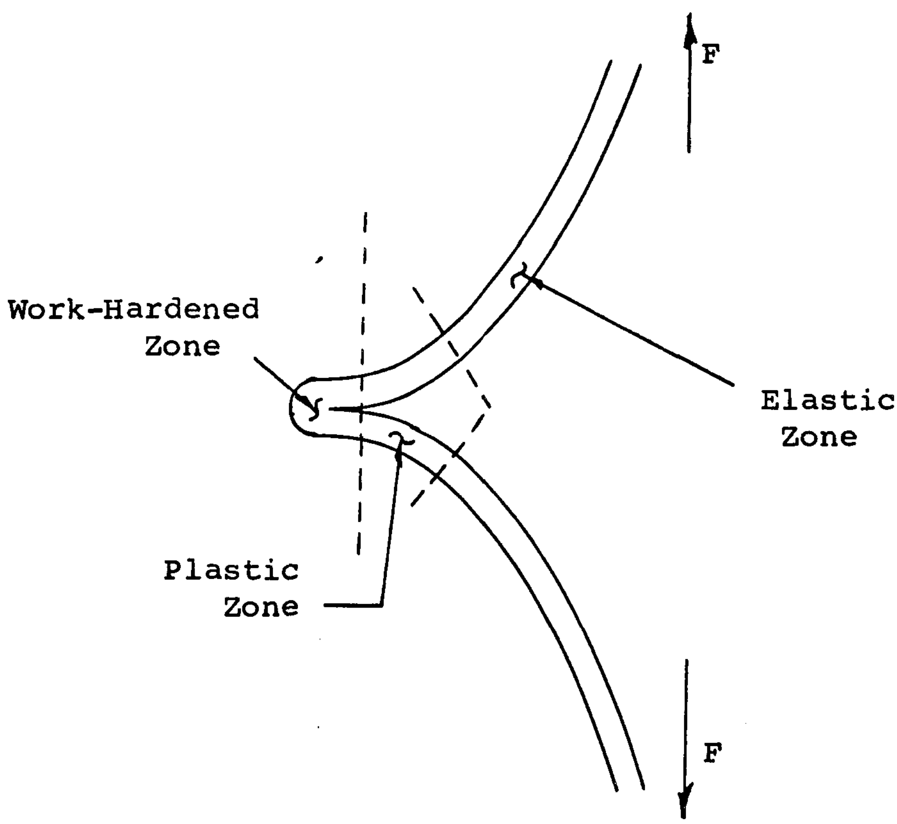

Figure 1. —

When plastic deformation is occurring in a region of the tape, the stress distribution is assumed to be as in figure 2.

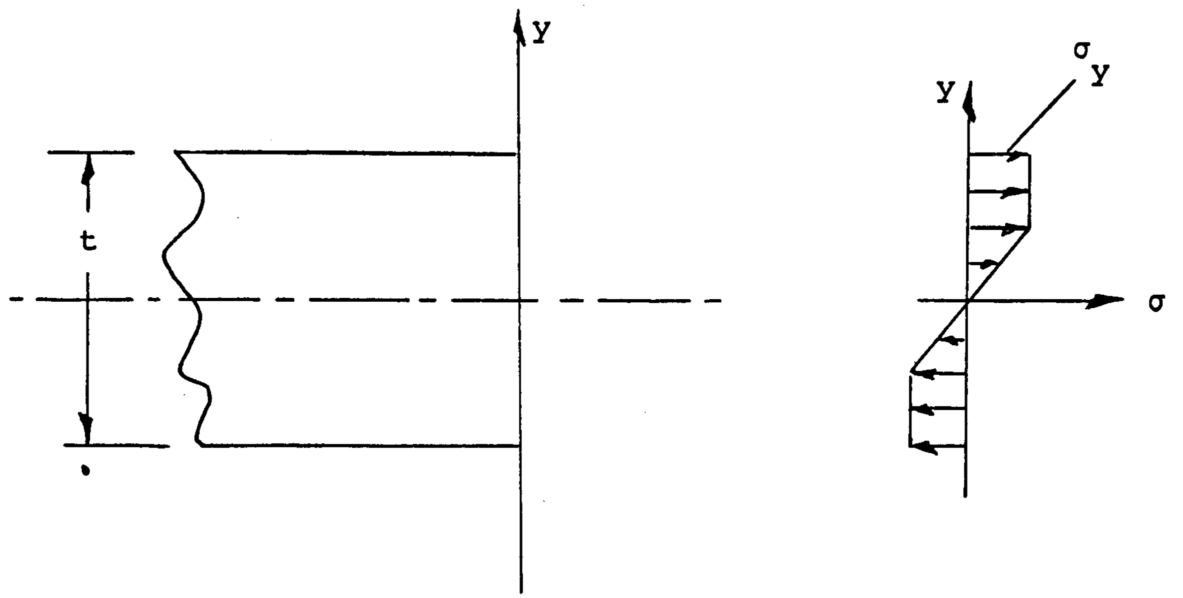

Figure 2. —

Before yielding occurs

$$M = 2\int_0^{t/2} \sigma_y \, dy = 2\int_0^{t/2} \frac{\sigma_{max}}{(t/2)} \cdot y^2 \, dy = \frac{\sigma_{max} t^2}{6} \qquad (1)$$

At the instant when yielding starts

$$M = \frac{\sigma_Y t^2}{6} \tag{2}$$

And in the limit, for a non-work hardening material, (when all material is stressed to $\pm \sigma_Y$)

$$M = 2 \int_0^{t/2} \sigma_Y y dy = \frac{\sigma_Y t^2}{4} \tag{3}$$

The actual moment range in which yielding occurs is not large since

$$\frac{\sigma_Y t^2}{6} < M < \frac{\sigma_Y t^2}{4} \tag{4}$$

An exact analysis of the shortening of the tape due to the crease is obviously rather complicated even if the effects of work hardening at the crease are neglected. It is therefore assumed that no yielding occurs for

$M < \frac{\sigma_Y t^2}{4}$ and that a plastic hinge occurs when $M > \frac{\sigma_Y t^2}{4}$

and the crease then opens up until the total included angle, $2\theta_o$, is such as to reduce the moment at the crease to

$$M_o = \frac{\sigma_Y t^2}{4} \tag{5}$$

where M_o is M at crease.

The force balance on the tape on one side of the crease is shown in Figure 3.

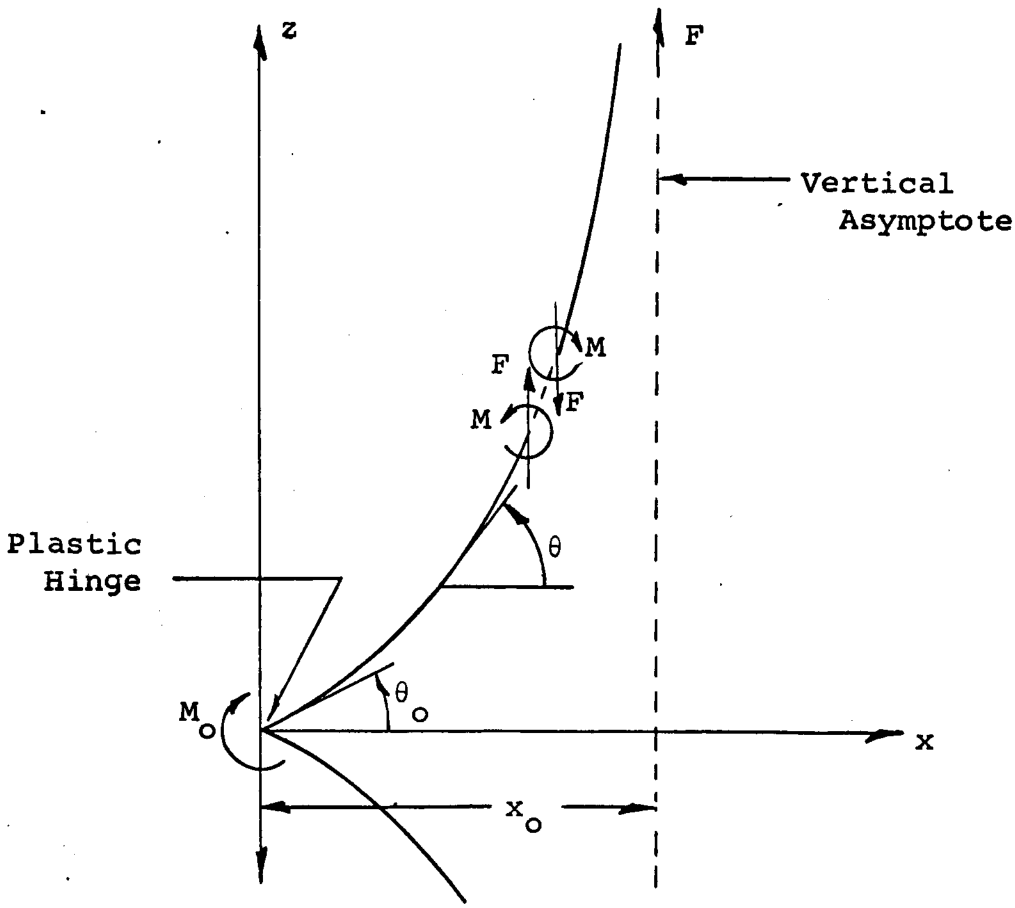

Figure 3. —

Where

$$M = F\left(x_o - x\right) \tag{6a}$$

$$M_o = Fx_o \tag{6b}$$

The curvature is given by

$$\frac{d\theta}{ds} = \frac{M}{EI} \tag{7}$$

where

$$\cos\theta = \frac{dx}{ds} \tag{8}$$

Substitution of (6) and (8) into (7) gives

$$\cos\theta \cdot d\theta = \frac{F}{EI}\left(x_o - x\right)dx \tag{9}$$

Integration of (9) between the origin and a general point yields

$$\int_{\theta_o}^{\theta} \cos\theta \cdot d\theta = \frac{F}{EI}\int_{o}^{x}\left(x_o - x\right)dx \tag{10a}$$

or

$$\sin\theta - \sin\theta_o = \frac{F}{EI}\left(x_o - \frac{x}{2}\right) x \tag{10b}$$

When $\theta = \frac{\pi}{2}$ (or when z becomes infinite), $x = x_o$ and

$$\sin\theta_o = 1 - \frac{F}{2EI}\cdot x_o^2 \tag{11}$$

Substituting (11) into (10b) yields

$$\sin\theta = 1 - \frac{F}{2EI}\left(x - x_o\right)^2 \tag{12}$$

SHORTENING CAUSED BY CREASE

Define $\Delta \ell$ as the extra length of the tape between two remote points on the tape because of the crease. Then

$$d\ell = ds - dz = \left(\frac{1}{\cos\theta} - \tan\theta \right) dx = \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} \cdot dx \qquad (13)$$

Now let

$$\xi = \left(x_o - x \right) \qquad (14a)$$

and

$$\xi_o = x_o \qquad (14b)$$

and substitute into (12) to give

$$\sin\theta = 1 - \frac{F}{2EI} \cdot \xi^2 = 1 - \frac{\xi^2}{\xi_h^2} = 1 - \bar{\xi}^2 \qquad (15)$$

where

$$\xi_h = \sqrt{\frac{2EI}{F}} = \text{value of } \xi \text{ when } \theta = 0$$

$$\bar{\xi} = \frac{\xi}{\xi_h}$$

$$\bar{\xi}_o = \frac{\xi_o}{\xi_h}$$

These new coordinates are shown in figure 4.

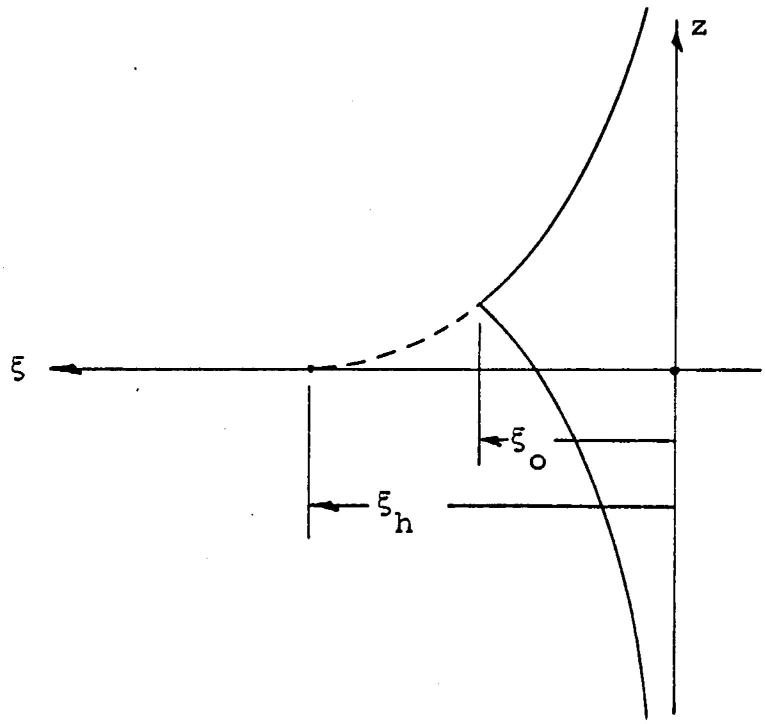

Figure 4. —

By integrating (13)

$$\Delta\ell = 2\int_{0}^{x_o} \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}\cdot dx = -2\int_{\xi_o}^{o} \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}\cdot d\xi = 2\int_{o}^{\xi_o} \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}\cdot d\xi \qquad (16a)$$

Substituting $\sin\theta$ from (15) yields

$$\Delta\ell = 2\xi_h \int_0^{\bar{\xi}_o} \frac{\bar{\xi}}{\sqrt{2 - \bar{\xi}^2}} \cdot d\bar{\xi} \tag{16b}$$

or

$$\Delta\ell = 2\xi_h \left[\sqrt{2} - \sqrt{2 - \bar{\xi}_o^2}\right] \tag{17}$$

where

$$\xi_h = \sqrt{\frac{2EI}{F}}$$

$$\bar{\xi}_o = \frac{\xi_o}{\xi_h}$$

$$\xi_o = \frac{M_o}{F}$$

$$\bar{\xi}_o^2 = \frac{M_o^2}{2EIF}$$

and

$$\sin\theta_o = 1 - \bar{\xi}_o^2 \tag{18}$$

Also

$$M_o = \frac{\sigma_y t^2}{4} \qquad \text{(from equation (5))}$$

$$F = \sigma_t t$$

$$I = t^3/12$$

σ_t = tensile stress away from crease

Then substituting the above into $\bar{\xi}_o^2$ yields

$$\bar{\xi}_o^2 = \frac{M_o^2}{2EIF} = \frac{3}{8}\cdot\frac{\sigma_y^2}{\sigma_t E} \quad \text{if a plastic hinge occurs} \tag{19}$$

$$\bar{\xi}_o^2 = 1 \qquad \text{for elastic deformation} \tag{20}$$

Substituting F and I into ξ_h gives

$$\xi_h = \sqrt{\frac{2EI}{F}} = \sqrt{\frac{Et^2}{6\sigma_t}} \qquad \text{(general)} \tag{21}$$

The boundary between plastic and elastic deformation can also be expressed as

$$\frac{\sigma_t E}{\sigma_y^2} < \frac{3}{8} \qquad \text{for elastic deformations} \tag{22}$$

$$\frac{\sigma_t E}{\sigma_y^2} > \frac{3}{8} \qquad \text{for plastic hinge} \tag{23}$$

and now by substituting (19), (20), and (21) into (18)

$$\Delta\ell = t\sqrt{\frac{E}{\sigma_t}}\cdot f\left(\frac{\sigma_t E}{\sigma_y^2}\right) \tag{24a}$$

or

$$\frac{\Delta \ell}{t\sqrt{\frac{E}{\sigma_t}}} = f\left(\frac{\sigma_t E}{\sigma_y^2}\right) \qquad (24b)$$

where

$$f\left(\frac{\sigma_t E}{\sigma_y^2}\right) = \sqrt{\frac{2}{3}}\left[\sqrt{2} - 1\right] \quad \text{for} \quad \frac{\sigma_t E}{\sigma_y^2} < \frac{3}{8} \qquad (25a)$$

$$f\left(\frac{\sigma_t E}{\sigma_y^2}\right) = \sqrt{\frac{2}{3}}\left[\sqrt{2} - \sqrt{2 - \frac{3}{8}\frac{\sigma_y^2}{\sigma_t E}}\right] \quad \text{for} \quad \frac{\sigma_t E}{\sigma_y^2} > \frac{3}{8} \qquad (25b)$$

The relationship expressed by equations (24) and (25) has been evaluated numerically and is shown in figure 5.

It can be seen from equation (21) or figure 4 that ξ_h describes the elastic behavior of the ribbon and is independent of whether a plastic hinge forms at the crease or not. On the other hand, $\bar{\xi}_o^2$ depends only upon the angle which the plastic hinge forms at the crease, as can be seen from equation (18). When applied to equation (24a) the above means that $t\sqrt{\frac{E}{\sigma_t}}$ describes the elastic curve, although not its point of termination at the crease, and $f\left(\frac{\sigma_t E}{\sigma_y^2}\right)$ describes the angle, θ_o, at the plastic hinge.

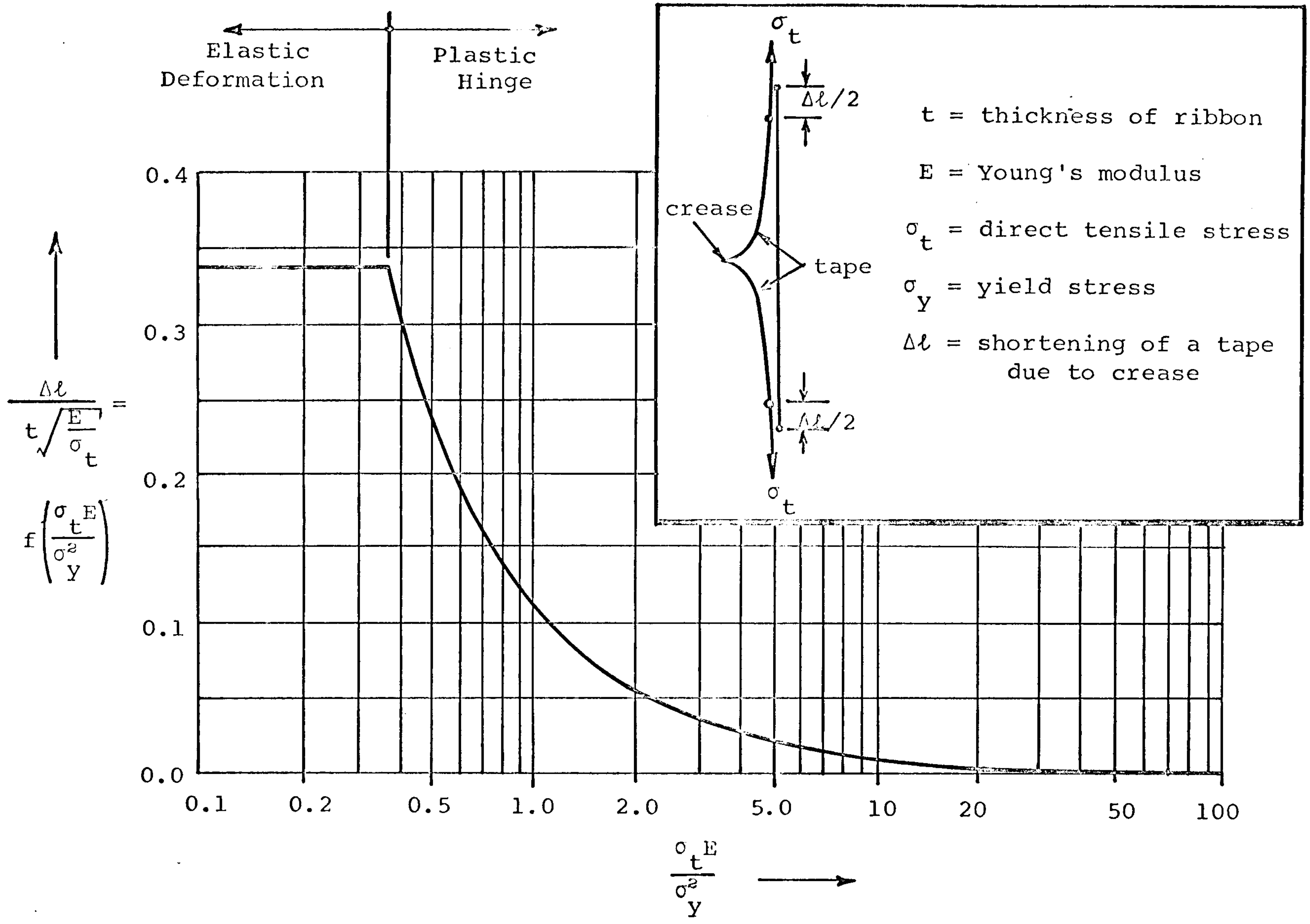

Figure 5. — Shortening of a Tape due to an Initial Crease

EFFECTIVE TENSILE MODULUS OF CREASED TAPE

Assume that the tape has previously been loaded to some value σ_{t_1} and is presently being loaded to a lesser value σ_{t_2}. Then according to the discussion on page 14, equation (24a) can be written as

$$\Delta \ell = t\sqrt{\frac{E}{\sigma_{t_2}}} \cdot f\left(\frac{\sigma_{t_1} E}{\sigma_Y^2}\right) \tag{26}$$

and

$$\frac{\partial(\Delta \ell)}{\partial \sigma_{t_2}} = -\frac{t}{2}\sqrt{\frac{E}{\sigma_{t_2}^3}} \cdot f\left(\frac{\sigma_{t_1} E}{\sigma_Y^2}\right) \tag{27}$$

The effective (or apparent) tensile modulus, E_{eff}, of a tape of length ℓ which contains one crease is then

$$\frac{1}{E_{eff}} = \frac{\partial \epsilon}{\partial \sigma_{t_2}} = \frac{\partial \epsilon_{elast}}{\partial \sigma_{t_2}} + \frac{\partial \epsilon_{crease}}{\partial \sigma_{t_2}} \tag{28}$$

where

ϵ = total apparent strain

$\epsilon_{elast.}$ = strain due to elongation of tape

ϵ_{crease} = apparent strain resulting from straightening of crease

Then

$$\frac{1}{E_{eff}} = \frac{1}{E} - \frac{1}{\ell} \cdot \frac{\partial(\Delta \ell)}{\partial \sigma_{t_2}} \tag{29}$$

or

$$\frac{1}{E_{eff}} = \frac{1}{E} + \frac{t}{2\ell} \sqrt{\frac{E}{\sigma^3_{t_2}}} \cdot f\left(\frac{\sigma_{t_1} E}{\sigma^2_Y}\right) \tag{30}$$

CONCLUDING REMARKS

It should be recognized that the foregoing analysis is based upon a greatly simplified model and does not include the effects of work hardening nor the detailed and complex stress distribution in the vicinity of the crease. An experimental investigation is being initiated to determine the actual tensile behavior of creased strips and will be reported at a future date.

Based upon the simplified model, the following conclusions can be made. The effective longitudinal compliance of a metallic tape with a transverse tape is composed of two parts, the compliance intrinsic to a straight strip and the compliance resulting from the "springiness" in the vicinity of the crease. It is found that a tape operating at a relatively low tensile stress can be made to have a minimum effective longitudinal compliance by choosing a material with a low yield stress (which allows plastic straightening at the crease), by choosing material with low tensile modulus (which allows elastic straightening in the vicinity of the crease), by using a minimum thickness of tape, and by operating at as high a stress as is feasible.

Astro Research Corporation
Santa Barbara, California, December 13, 1966.